

Surplus Modelling in PillarOne

Martin Melchior, Michael Noé

March 9, 2009

Preamble

In this part of the documentation, we give a description of how surplus reinsurance is modeled in PillarOne including the components that can be used, their parameterization and how they can be used from within the graphical user interface.

Contents

1	Select the type of the Risk Allocator	11
2	Risk Band Claims Generators	11
3	Standard Claims Generators and Risk Allocators	11
3.1	Risk Allocation for Aggregate Claims	12
3.2	Risk Allocation for Single Claims	12
4	Standard claims generator and sum insured / PML	14
4.1	Allocation of aggregated claims	14
4.2	Generating the SumInsured for Single Claims	14

Introduction Surplus reinsurance is a form of proportional reinsurance. In contrast to quota share reinsurance, the portion covered by the reinsurer depends on the sum insured¹ specified for the underlying insured risks. All the risks with sum insured up to a certain line are fully covered by the reinsured - the exceeding part up to a certain #lines is then covered by the reinsurer. The portion of the *claim ceded* to the reinsurer is given by the ratio of the *risk premium* divided by the *total premium*. As a result, the modeling of surplus reinsurance requires additional information - a manifest link of the claims to the underlying risks. We will give details on how this link can be provided. Note that for the other reinsurance treaty types currently modeled in PillarOne no such link is needed.

The integration of surplus reinsurance into the component-based modelling framework of PillarOne attempts to design the components as general as possible. The goal is to reuse the components as far as possible - when new models are constructed or the functionality provided by the modeling framework needs to be extended. In this sense, the link between claims and risk information is modeled in dedicated components - while the information is used in a generic surplus reinsurance component. We see two different approaches to model the link between risk and claims information:

- Attaching the risk information to the risks before drawing the claims
- Attach risk information while generating the claims: Typically, start with the risk information and use risk-dependent characteristics for simulating the claims.
- Attach risk information after generating the claims: Here, we can start with standard claims generators and try to map the claims to the risks such that suitable characteristics on the risk portfolio are met.

Generally, we consider the first approach more sound and we suggest using this whenever possible. However, more detailed data (including claims per risk or per risk class statistics) are needed for a calibration of such risk-dependent claims generators. Therefore, we opt for the second approach in case not all data are available for a calibration of risk-dependent claims generators. Here, suitable 'RiskAllocator' components are used which are typically based on more general risk portfolio characteristics.

For illustration purpose we prepared a simple surplus model which is shipped with the basic PillarOne release. After starting up PIRAT select the model 'OneLobSurplus' and have a look at the parameters by double-clicking one of the parameter sets under 'Parameterization' (as shown in Figure [2](#)). In this model, the second approach for modeling the claims with risk information is adopted.

¹or other quantities used to characterize the underlying risk such as 'probable maximum possible' (PML).

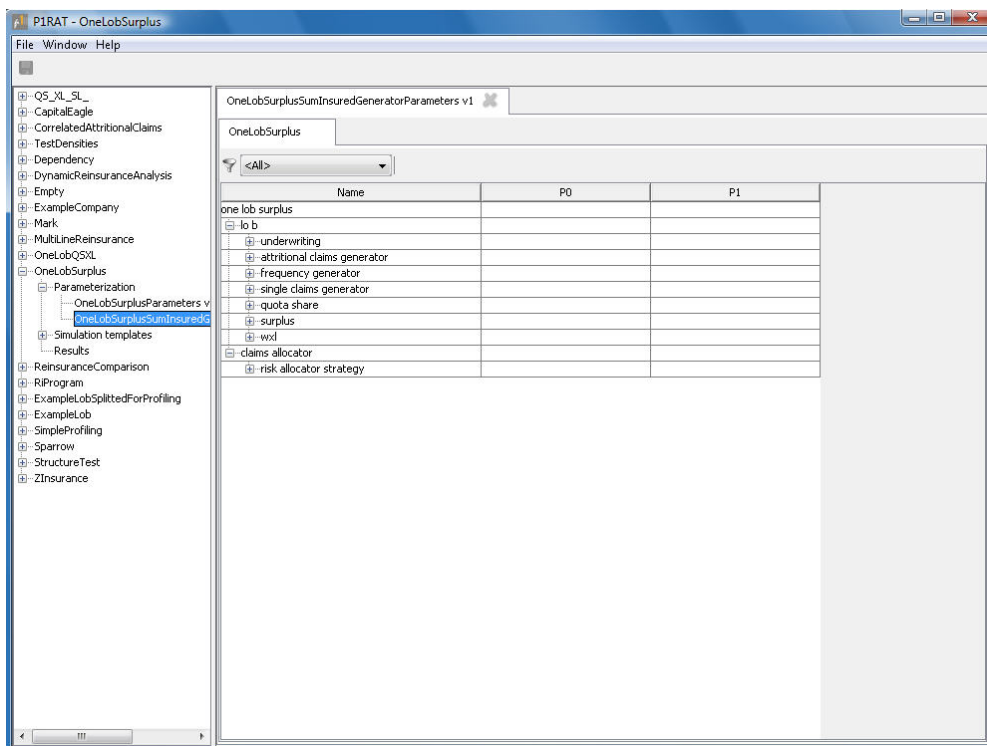


Figure 1: Choose the 'OneLobSurplus'-model which illustrates the basic approach for modeling surplus reinsurance.

fig: select-surplus-m

Surplus Reinsurance Component The mechanics of a surplus reinsurance contract is included in a model by using a general ReinsuranceContract component and setting the contract type equal to 'Surplus'. Figure shows how to select the contract type from within the GUI. This makes the general reinsurance contract component behave like a surplus contract.

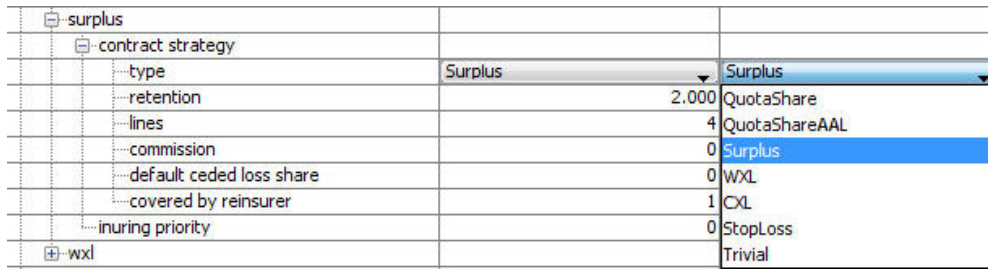


Figure 2: Set the contract type to 'Surplus' so that the component behaves like a surplus contract.

fig:select-surplus-c

In addition, the surplus-specific parameters appear on the screen:

- *Retention* (v_0) of the surplus contract. Use units consistent with the units used for the sum insured (see Section [sec:generating-claims-with-exposure](#) below).
- Number of *lines* (m) the contract covers - hence the contract covers up to a limit of $m \cdot v_0$.
- The *commission* the contracts pays to the reinsured – as a percentage of gross premium.

For a risk with sum insured v the proportion of the claim ceded to the reinsurer is then given by

$$\rho(v_i) = \frac{\min(m \cdot v_0, (v_i - v_0)_+)}{v_i} \quad (1)$$

eq:fraction-ceded-su

As the general reinsurance component, the surplus component (i.e. the reinsurance component with surplus behavior) accepts lists of claims as input. The result of the surplus is non-trivial only for claims that have risk information attached. For each claim, this risk information (v_i) is used to compute the fraction ceded given by the above formula [\(I\)](#).

If a loss has no sum insured / PML after allocation, then it's typical an attritional loss, and will be ceded like described in equation [1](#), with S_0 is the attritional loss and S_0^{RI} is the part which will be paid by the reinsurer. For these attritional losses typically a small share is reinsured under the surplusshare RI treaty. The exact share cannot be determined

$$S_0^{RI} = \alpha \cdot \beta \cdot S_0 \quad (2)$$

equ:ceded attritiona

Known restrictions:

- At the moment, different retentions specified for different claims types can only be modeled by using different surplus components and allocating the associated claims accordingly. Different claims types and different retentions should be considered when setting up the model. However, work on generalizing the component to allow for specifying different retentions for different claims types is in progress.

Underwriting Component In the underwriting component(s), the data used to characterize the portfolio of policies is specified. Typically, the insurers report on the details of the underwriting portfolio by grouping individual risks or policies with similar sum insured in suitable classes. These classes can be characterized by sums insured lying in specific ranges.

maximum sum insured	average sum insured	premium	number of policies/risks
1000.0	800.0	50000.0	1000
10000.0	8000.0	50000.0	100
100000.0	80000.0	50000.0	10

Figure 3: Specify the characteristics of the underwriting portfolio.

fig:underwriting-info

In our illustration model go to 'underwriting' and double-click the 'underwriting information' data. As a result, a table open in a new tab, in which these risk classes are organized as different rows (see Figure 3). The following columns are included in the table:

- 'maximum sum insured': The upper bound of the range of sums insured. The lower bound is given by the upper bound of the previous class (in the previous row) or 0.
- 'average sum insured': The average sum insured of the policies included in the given risk class. By definition, the average should be smaller than the maximum sum insured of the given row, but larger than the maximum sum insured of the previous row.
- 'premium': The total premium of all the policies included in the class.
- 'number of policies': The number of policies included in the class.

Note that the upper bound of the ranges should be in increasing order. The number of risk bands can be modified by changing the rowCount variable and press Apply.

In the example depicted in Figure 3, there are three classes:

- The first class includes policies with sum insured between 0 and 1000,
- the second class has policies with sum insured between 1000 and 10'000 and
- for the third the policies lie in the range between 10'000 and 100'000.

In the table, two other 'custom' columns are appended, the 'custom allocation single claims' and the 'custom allocation attritional claims'. These columns are particularly useful in combination with the risk allocator component described in the next section. Similarly, the additional elements in the 'underwriting' section of the parameter set ('allocation base attritional claims', 'allocation base single claims') specify the information to refer to when allocating claims.² We will give some more explanation in the next section.

² From a logical point of view, these parameters should rather be included in the allocator component. They are included at this level due to practical reasons.

claims-with-exposure

Generating Claims with Exposure Information In this section, we describe how claims with exposure information attached can be generated. As mentioned above, there are (at least) two approaches to generate such claims,

- using a generator that is based on risk dependent claims statistics ('risk band generators') or
- using standard claims generators and allocating the claims to suitable risk classes.
- generating a sum insured for single loss

1 Select the type of the Risk Allocator

In figure 4 you can see, how you choice your preferred type of RiskAllocators. The "NONE"-allocator does nothing, the "RiskToBand"-Allocator is described in section 3 and last but not least the "SumInsuredGenerator" is described in section 4.

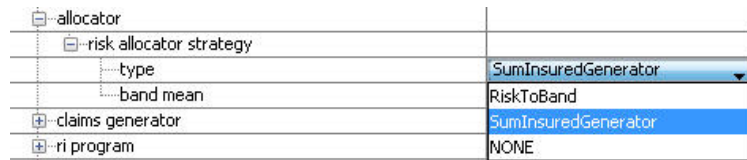


Figure 4:

fig:choice RiskAlloc

2 Risk Band Claims Generators

TODO

3 Standard Claims Generators and Risk Allocators

The idea here is to allocate the claims generated by a standard claims generator to risk classes. The information about its risk class can then be attached to the claims and subsequently used for instance in a surplus reinsurance component.

In principle, there are many different ways of allocating claims to risk bands. We here present two different approaches, one which can be applied to aggregate losses the second one which can be used for single losses. In both approaches, the allocation uses a target distribution that specifies for each risk class the percentage of claims that should be associated with the given risk class. In the case of aggregate claims, the target allocation can always be easily met since aggregate claims can always be split up. In the case of single claims, however, this

sec:RiskAllocation

is not possible since we cannot split up individual claims. Furthermore, for single claims the claim size is limited to the upper boundary of the risk class³ therefore the allocation to the target distribution is always conditioned on constraints.

For the further discussion, let us denote the target distribution by the numbers $n_k (1 \leq k \leq m)$ that are the proportions of the claims to be allocated to risk class k . These numbers are not necessarily normalized but should all be non-negative.

3.1 Risk Allocation for Aggregate Claims

Here, each incoming input loss of type 'aggregate' or 'attritional' is split up in m contributions where m is the number of risk classes. The amount allocated to class k is proportional to n_k , i.e.

$$C_k^{(\text{attr})} = n_k \cdot C^{(\text{attr})} / n \quad (3)$$

where $n = \sum_{k=1}^m n_k$. From N input claims $m \cdot N$ output claims are calculated. The corresponding risk class are attached to the claim so that this information is available in subsequent components (e.g. surplus reinsurance). Claims of type different than 'attritional' or 'aggregate' are not allocated - i.e. no risk class information is attached to these claims.

3.2 Risk Allocation for Single Claims

For an initial guess, the incoming (individual) losses are allocated to the risk classes by just referring to the claim size. To be specific, a single loss $C^{(j)}$ is mapped to risk class k if

$$v_k^{(\min)} \leq C^{(j)} \leq v_k^{(\max)} \quad (4)$$

The target allocation specifies for each risk class a target number of (single) claims. Hence, the initial guess is obtained as the number of claims obtained by [\(5\)](#), i.e. [eq:initial-guess-single-claims-allocation](#)

$$N_k^{(0)} = \left| \left\{ v_k^{(\min)} \leq C^{(j)} \leq v_k^{(\max)} \mid 1 \leq j \leq N \right\} \right| \quad (5)$$

In a second step, claims from a given band are reallocated to a higher band such that ideally the resulting allocation of the claims follows the target distribution. The reallocation is processed by starting with the uppermost risk class (where no reallocation is possible) and iteratively proceeding to lower classes. For class $k = m, m-1, \dots$, the exceeding number of claims

$$\Delta N_k^{(0)} = \left(N_k^{(0)} - \frac{n_k \cdot N}{n} \right)_+ \quad (6)$$

is allocated to upper classes in proportion to the number of claim needed to arrive at the number number of claims:

$$\Delta N_l^{(m-k)} \propto - \left(N_l^{(m-k-1)} - \frac{n_l \cdot N}{n} \right)_- \quad (7)$$

³Disregarding e.g. PML underestimation

with $l = k + 1, \dots, M$. Note that the numbers $N_l^{(m-k+1)}$ are obtained in the previous iteration step.

The order in which the claims are included in the output list is different from the order of the claims in the input list.

In both cases discussed above, the component RiskAllocator can be used – for aggregate claims allocation the 'claim type' parameter should be set to 'attritional', for single claims allocation it should be set to 'single'.

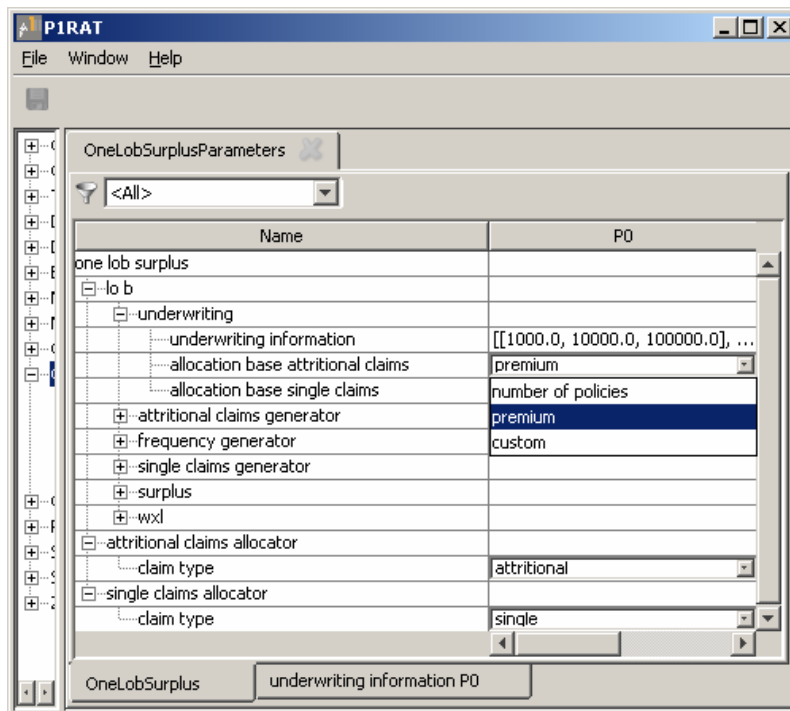


Figure 5: Selection of the target allocation for mapping claims to risk classes.

fig:target-allocation

In our example used for illustration purpose, the selection of the target distribution used for the allocation is specified in the underwriting component (see also footnote 2). In the example, 'premium', 'number of risks' and 'custom' can be selected. To give an example, when choosing 'premium' the target allocation quantities are set according to $n_k := P_k$. This would be a reasonable choice for aggregate claims whereas for single claims the number of risks probably would be a better choice. Using the columns 'allocation base attritional claims'

and 'allocation base single claims' in the underwriting component a custom target allocation can be specified for single and aggregate claims.

4 Standard claims generator and sum insured / PML

From the actuariell model, losses will be generated for every risk. For further handling, sum insured / PML will be identified for every loss.

4.1 Allocation of aggregated claims

The aggregated claims won't be splitted, to the RiskBands, like in section [3.1](#). They are allocated based on an preselected attritional loss ceeding ratio. sssec:Risk Allocation Aggregate Claims

4.2 Generating the SumInsured for Single Claims

Typical, we would locate, which policy is affected by a loss and identify the PML of this policy. Then the loss can be dedicated to the appropriate policy. In the collective modell the losses (here X_i) will be drawn independent from their policies and later a sum insured / PML named \mathbf{MIL}_i (maximum insured loss of the i^{th} loss) will be allocated to them. The \mathbf{MIL} without index is the upper bound of the supreme riskband.

We generate the sum insured / PML for each single claim, how it's described in equation [4.2](#). δ is a random variate. equ:estimate MIL

$$\mathbf{MIL}_i = X_i + \delta \cdot (\mathbf{MIL} - X_i) \tag{8} \quad \text{equ:estimate MIL}$$

The parameters of this method are the distribution of δ . This should be choosen, to become a delta between 0 and 1.

To fit a distribution to the random variate δ we propose the following method. The estimation starts with the statistics of δ .

$$\delta_i = \frac{\mathbf{MIL}_i - X_i}{\mathbf{MIL} - X_i} \tag{9} \quad \text{equ:estimate delta}$$

$$\mathbf{MIL} = \max_i \{\mathbf{MIL}_i\} \tag{10} \quad \text{equ:estimate mil}$$

For parameter estimation of the random variate families with mean near zero and support $[0; 1]$ is required. If the generated loss is bigger then the \mathbf{MIL} then the dedicated sum insured / PML will artificial set on \mathbf{MIL} .