Modeling Dependency in P1.RiskAnalytics

Tools needed for a Best Practice

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Agenda

- Setting up the scene: Dependency modeling within your company?
- Modeling approaches and capabilities in P1.RiskAnalytics
- Discussion: Further requirements / future work

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Why Modeling Dependency?

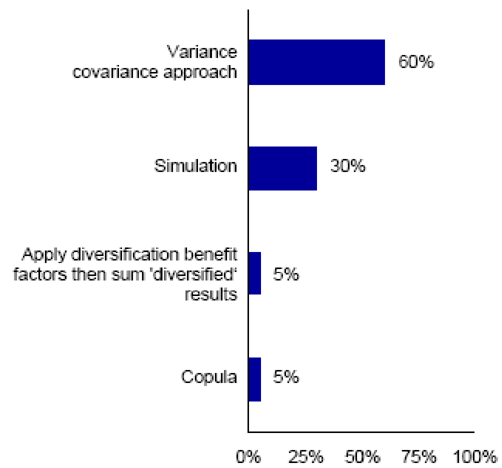
- Is part of best practice in (insurance) risk management:
  Two examples:
  - Accumulation control – typically catastrophe perils
    (perils that lead to dependent claims within portfolio of risks and across lines of business).
  - Scenario analysis
    - e.g. impact of rise in inflation by 3%
      (changes accumulate across different lines of business)
    - other market risk factors
      (correlated impacts also on the expenses and revenues side)

  → Evaluation of risk management strategy.
- Required for Solvency II

Industry Practice in Dependency Modeling: Study on Internal Models presented @CRO Forum

Majority of companies are currently using variance/covariance for overall aggregation approach, but there is a slight trend towards alternatives

Overall aggregation approach used in EC model[1]



- 60% of companies use variance/covariance for overall aggregation approach
- However this includes widespread use of stochastic models for significant risk types (DFA for P&C, stochastic models for market risks)
- Three companies using variance/covariance approach are considering alternatives
  - 2 considering switch to copula approach
  - 1 considering switch to Monte Carlo simulation based on Gaussian copula approach

1. Two companies used more than one overall aggregation approach (hence percentage for individual company is 5%)

Source: www.croforum.org

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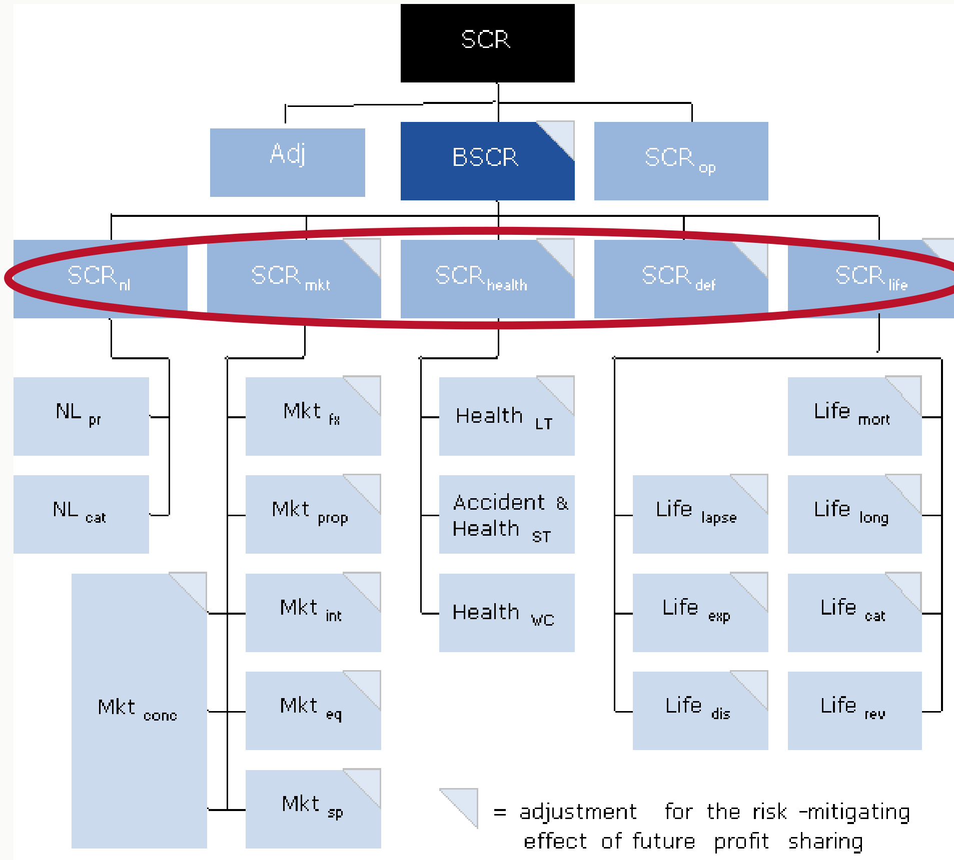
Dependency Models vs Modeling Techniques

Dependency Modeling Implicit in Solvency II Correlation Coefficients

Simple aggregation formula at different levels, e.g. the basic BSCR is computed according to

$$BSCR = \sqrt{\sum_{r,c} corrSCR_{r,c} \cdot SCR_r \cdot SCR_c}$$

CorrSCR	SCR_{mkt}	SCR_{def}	SCR_{life}	SCR_{health}
SCR_{mkt}	1			
SCR_{def}	0.25	1		
SCR_{life}	0.25	0.25	1	
SCR_{health}	0.25	0.25	0.25	1
SCR_{nl}	0.25	0.5	0	0.25



Source: QIS4 Technical Specifications

Dependency Modeling:
Different Approaches at Different Levels

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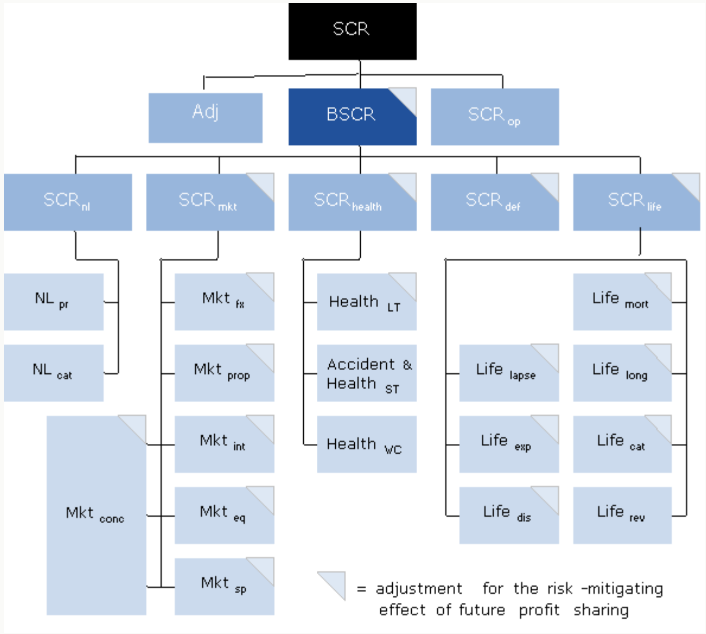
At different levels

Dependencies between

- risk categories at overall company level (non-life, life, health, credit, market)
- risk types within the risk categories
- within risk types between LoB's, perils, etc
- other (?)

Different modeling approaches

- simple (aggregation) formula
- copula: Parametric, independent of marginal distributions
- common cause: Model structure + "sensitivity functions"
- other (?)



General Rule:
A (dependency) model should be
- as simple as possible
- as complex as needed

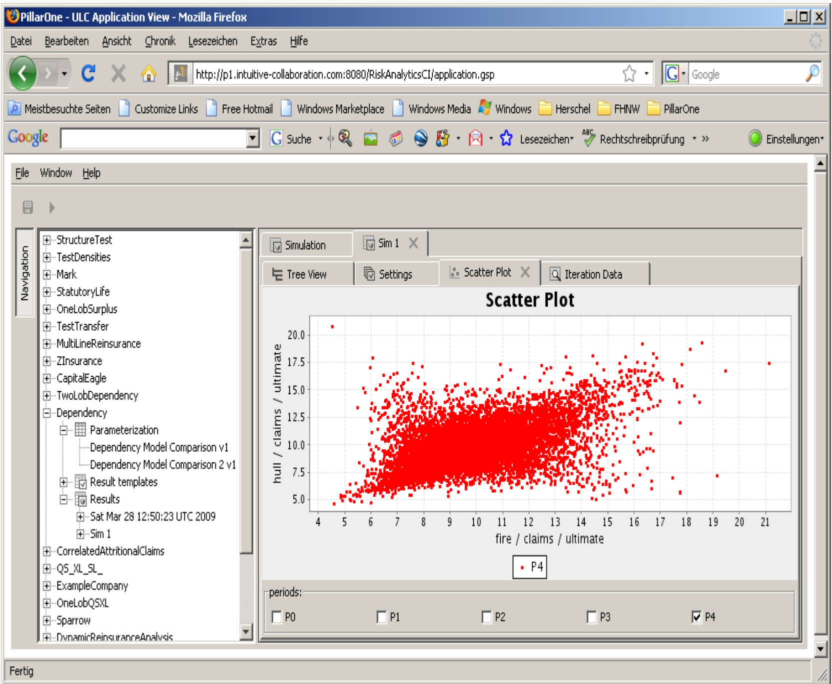
Modeling Dependency in Your Company?

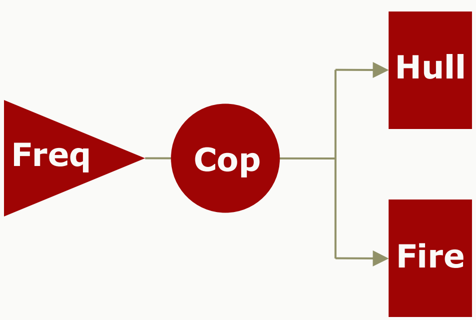
	Used Approach			
	Simple Formula	General Copula	Common Cause	Other (Var-CoVar)
risk categories at overall company level	**40%**	**15%**	**15%**	**30%**
risk types within risk categories	**25%**		**8%**	**67%**
within risk types between LoB's, perils, etc	**23%**	**23%**	**8%**	**46%**
other	**67%**			**33%**

Agenda

- Setting up the scene: Dependency modeling within your company?
- Modeling approaches and capabilities in P1.RiskAnalytics
    - Non-life insurance claims modeling based on copulas
    - Examples for common cause modeling
- Discussion: Further requirements / future work

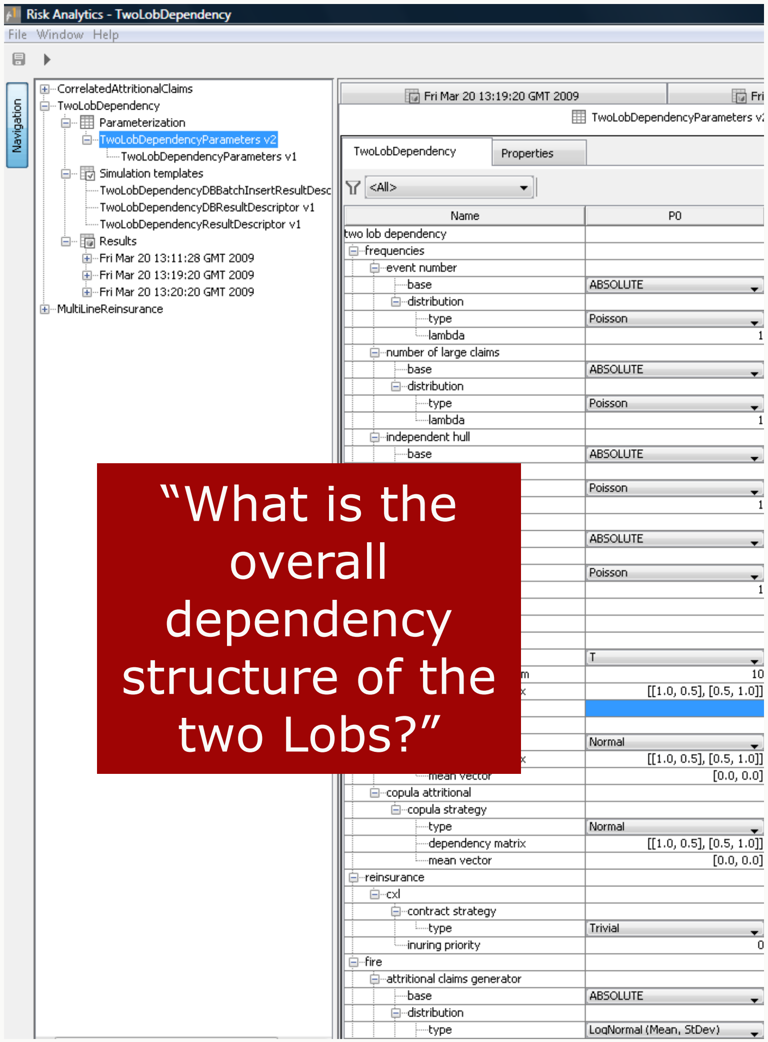
Modeling Dependency Matters (?)





	Mean	Stdev	VaR 99%	Corr Coeff
Marginal Distributions				
(Lognormal, Fire and Hull)	10.0	2.0	5.5	
Aggregate Claims				
Aggregation formula	20.0	n.a.	9.6	50%
Normal copula	20.0	3.4	9.2	50%
T-Copula (1)	20.0	3.4	9.6	50%
T-Copula (2)	20.0	3.4	10.1	50%
Co-monotone	20.0	4.0	11.0	100%

A “Typical” Claims Model



“What is the overall dependency structure of the two Lobs?”

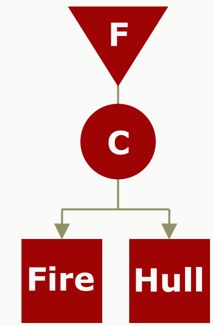
Two Lines of Business (Property, Motor Hull) with exposure to cat perils:

Event Losses

FrequencyGenerator(Fire,Hull)	Poisson(1.0)
Copula(Fire,Hull)	T(70%,3)
Marginal(Fire)	PARETO(1000,1.2)
Marginal(Hull)	PARETO(1000,1.2)

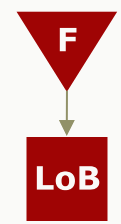
Dependent Large Losses

FrequencyGenerator(Fire,Hull)	Poisson(1.0)
Copula(Fire,Hull)	Normal(50%)
Marginal(Fire)	PARETO(500,1.5)
Marginal(Hull)	PARETO(500,1.5)
Truncated at 20000	



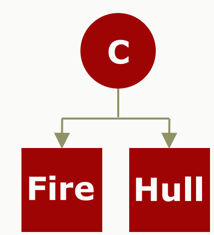
Independent Large Losses

FrequencyGenerator(Hull)	Poisson(10.0)
FrequencyGenerator(Fire)	Poisson(10.0)
Marginal(Fire)	PARETO(500,1.5)
Marginal(Hull)	PARETO(500,1.5)
Truncated at 2000	



Dependent Attritional Losses

Copula(Fire,Hull)	Normal(50%)
Marginal(Fire)	LN(100000,6000)
Marginal(Hull)	LN(100000,6000)



How Dependency Assumptions Become Manifest in Different Risk Measures

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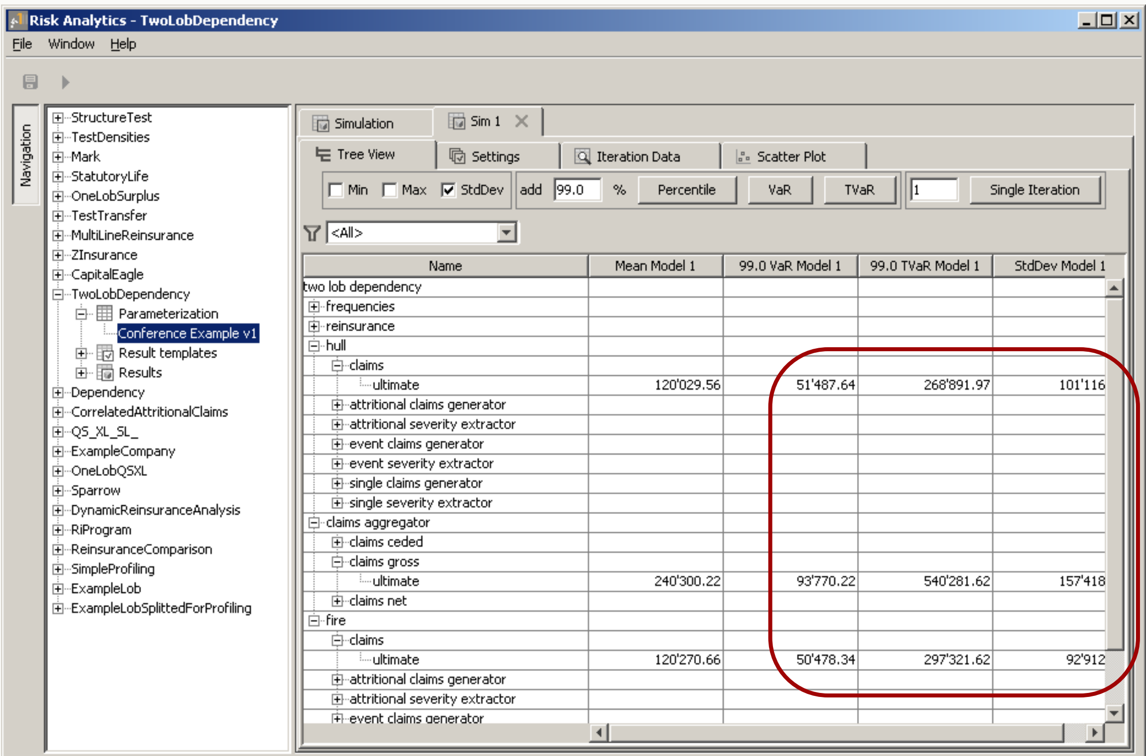
Risk Management meets Open Source

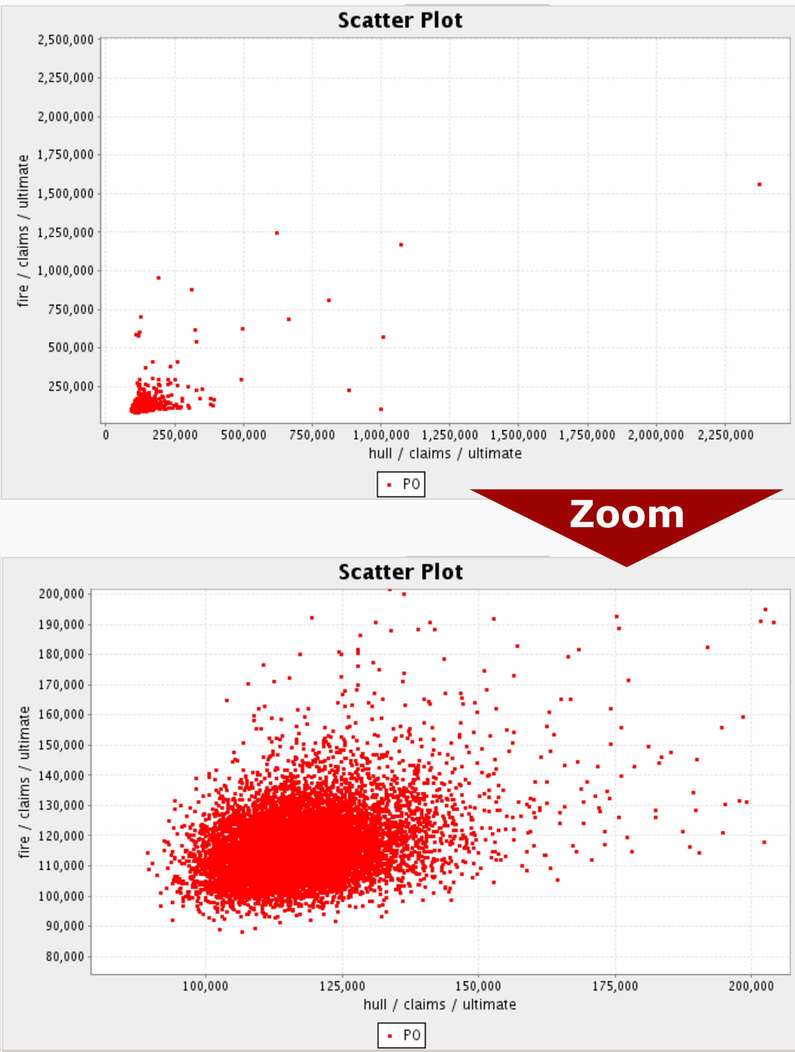


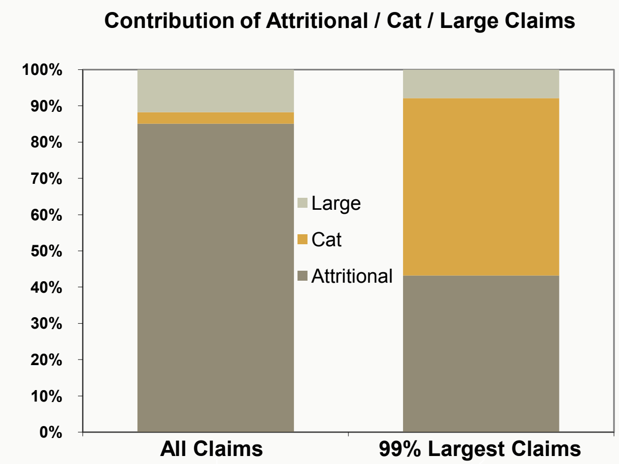
Diversification Benefit:

StdDev	**19%**
VaR (99%)	**8%**
TVaR (99%)	**5%**

The diversification benefit for measures localized in the tail of the distribution is much smaller than for measures that are rather localized in the bulk of the distribution. The resulting aggregate claims exhibit tail dependency.

Modeling Dependency Matters!





The diversification (or its lack) in the tail of the aggregate claims distribution is mostly driven by cat events and its assumptions on the dependency of its contributions to Fire and Hull.

Stochastic tools needed for modelling dependencies for the risk aggregation.

Non-Life Insurance in RiskAnalytics
More Detailed Claims Model (3)

	Property Buildings / Property Contents	
Aggregate Attritional Claims	▪ Aggregate claims ▪ Compound claims models aggregated	▪ Copula ▪ Common causes (eg inflation)
Single Large Claims	▪ Frequency-severity modeling	▪ … events: 'Correlated' …requency; for these, 'correlated' severity ▪ Common causes (eg inflation)
Cat Claims	▪ Event Loss Tables ▪ Frequency-Severity Distributions	▪ Identification by eventid ▪ Copula ▪ Common causes (eg inflation)

Dependencies across the different claims types?

Marginal Distributions

Dependency Modeling

Rather Common Cause: Catastrophe Perils Modeling

UNIQUE _ID	RATE	exploss	stdloss	exploss_F	exploss _F%	CoV	FAC
4_8/6_2	4.64E-05	-	-	-	1.0	4.0	0.0
4_8/6_3	2.60E-05	-	-	-	1.0	4.0	0.0
4_8/6_4	1.46E-05	12'307'086	11'407'983	12'316'668	1.0	4.0	0.0
4_8/6_5	8.20E-06	249'873'572	235'377'507	247'895'493	1.0	4.0	0.0
4_8/7_1	8.18E-05	-	-	-	1.0	4.0	0.0
4_8/7_2	4.59E-05	-	-	-	1.0	4.0	0.0
4_8/7_3	2.57E-05	-	-	-	1.0	4.0	0.0
4_8/7_4	1.44E-05	24'181'850	22'535'875	24'200'677	1.0	4.0	0.0
4_8/7_5	8.11E-06	319'421'797	301'213'133	316'480'786	1.0	4.0	0.0
4_8/8_1	6.08E-05	-	-	-	1.0	4.0	0.0
4_8/8_2	3.41E-05	-	-	-	1.0	4.0	0.0
4_8/8_3	1.92E-05	-	-	-	1.0	4.0	0.0
4_8/8_4	1.07E-05	33'696'970	31'466'356	33'591'111	1.0	4.0	0.0
4_8/8_5	6.03E-06	350'861'649	330'916'900	347'799'424	1.0	4.0	0.0
4_8/9_1	8.97E-05	-	-	-	1.0	4.0	0.0
4_8/9_2	5.04E-05	-	-	-	1.0	4.0	0.0
4_8/9_3	2.83E-05	1'670'204	1'563'814	1'671'505	1.0	4.0	0.0
4_8/9_4	1.59E-05	47'210'669	44'224'776	47'014'218	1.0	4.0	0.0
4_8/9_5	0.00E+00	-	-	-	1.0	4.0	0.0
4_8/10_1	9.42E-05	-	-	-	1.0	4.0	0.0
4_8/10_2	5.28E-05	-	-	-	1.0	4.0	0.0
4_8/10_3	2.97E-05	-	-	-	1.0	4.0	0.0
4_8/10_4	1.66E-05	41'020'984	38'449'501	40'945'536	1.0	4.0	0.0
4_8/10_5	0.00E+00	-	-	-	1.0	4.0	0.0
4_8/11_1	9.68E-05	-	-	-	1.0	4.0	0.0
4_8/11_2	5.43E-05	-	-	-	1.0	4.0	0.0

- Natural perils events are common causes that may cause losses in different lines of business, different legal entities.
- Commercial natural perils tools provide physical models for the events and their impacts on the portfolio(s).
- Use the "Event Loss Tables" (e.g. for each legal entity) to draw events - use unique id to identify the events and their losses in the different portfolios.
- How to 'correlate' the 'secondary uncertainties'?

Can easily be integrated in P1.RiskAnalytics – but has not yet been done.

Common Cause Modeling: Some Examples

Causes modeled as risk factors	Exposure of products to risk factors
Common risks lead to 'dependent' outcomes.	Sensitivity functions: How do cash flows or values depend on risk factors?
Interest rate risk	F/I securities, etc
	Lapse and surrender rates of policyholders in life insurance for different products
Inflation	Super-imposed claims inflation
	Inflation clauses in (re-)insurance products
Natural perils	Exposure of insurance policies to natural perils
etc	etc

Copula vs Common Cause: Some Pros and Cons

Copula

+ Way to express dependency independent of marginal distributions; marginal distribution is not distorted by the dependency modeling.

± Data-centric: Believe what the statistical tools see in the data.

− Hard to get an intuitive understanding?

Restriction to a few well known models?

Common Cause

+ Understanding of the world is (needs to be) reflected in the model. Intuitive.

− Depending on how the sensitivity functions are defined, dependency model no longer independent of marginal distributions; marginal distribution may be distorted by dependency structure.

− What sensitivity function? More or less intuitive.

Restriction to some well-defined sensitivity functions.

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Discussion

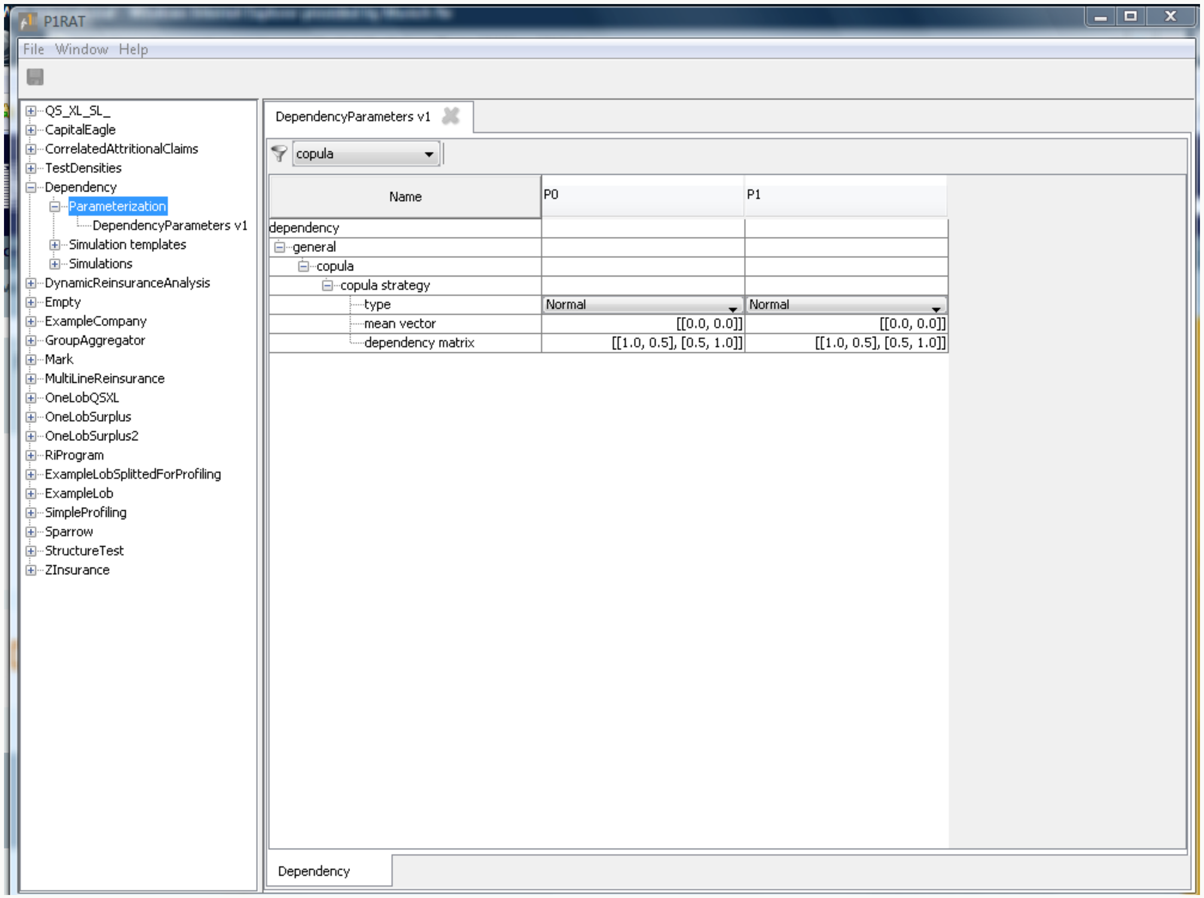
- Community approach - is it realistic? Does it make sense?
- How would you define a best practice approach?
- Further techniques to model dependencies
  - Further specific models (e.g. copula models, generators for common causes)
  - Sensitivity functions
  - Simulation techniques, e.g. simulating rank order correlation.
- Advanced model building capabilities?

Contributions to P1.RiskAnalytics are welcome

- Contribute at the Pillar1 forum
- Test P1.RiskAnalytics as a user and give feedback
- Start working with P1.RiskAnalytics (hey, it is open source!) and also make your code publicly available to the community.

Backup

Modeling Capabilities in RiskAnalytics: Copula



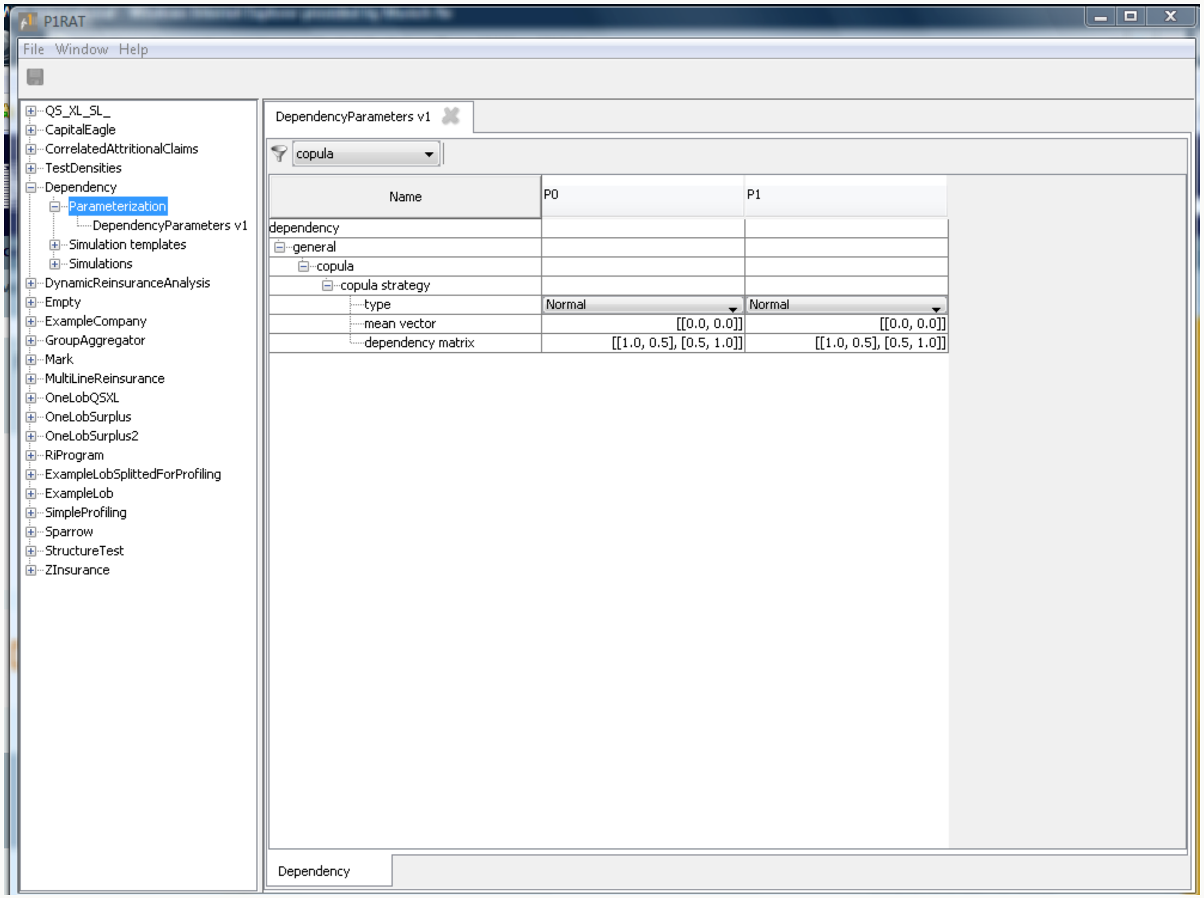
General Structure:

- Dependency structure defined at model specification time. Parameterization from within GUI.

Available models:

- Normal-copula
- t-copula
- Frechet upper bound
- independent

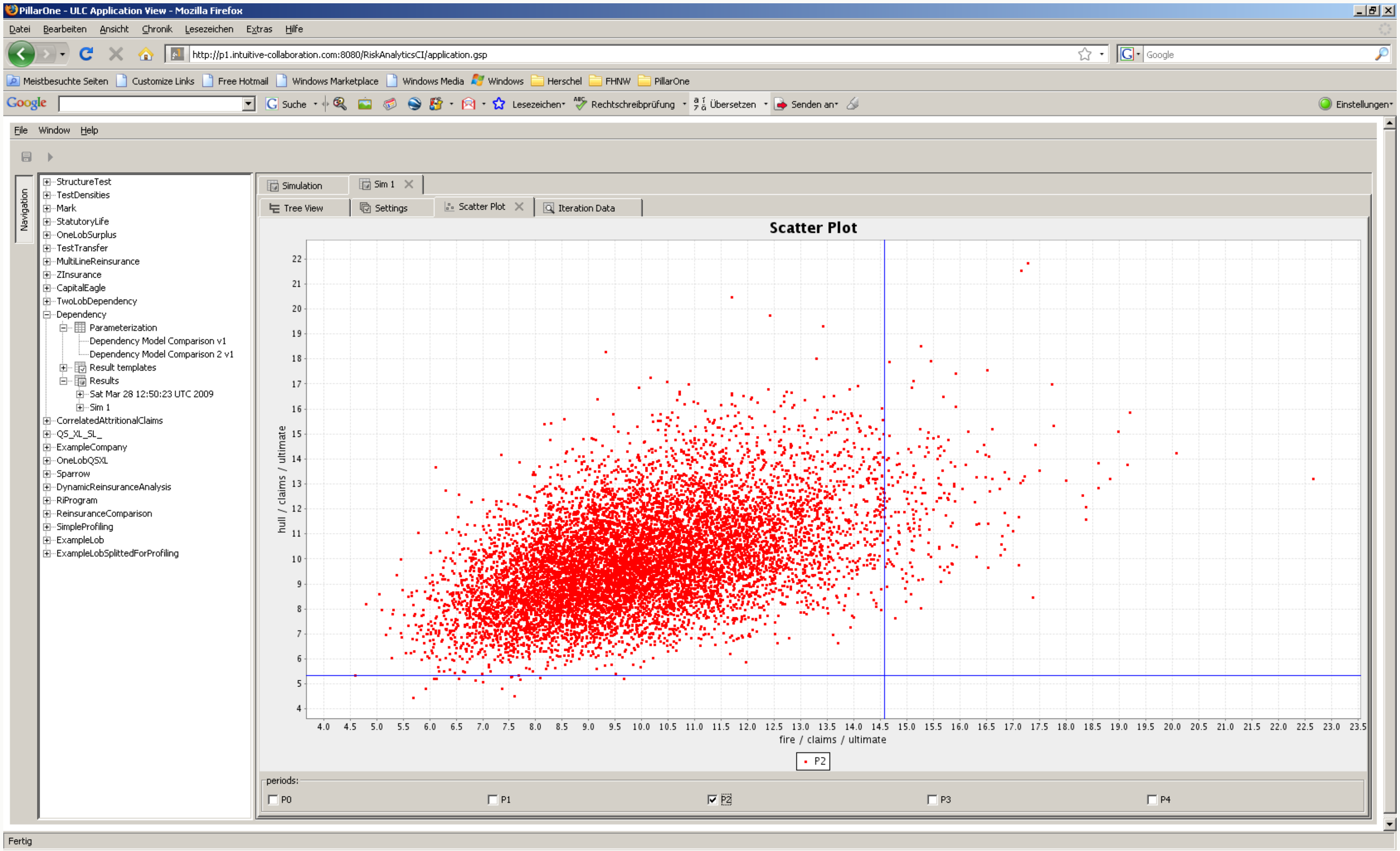
Modeling Capabilities in RiskAnalytics: Common Cause



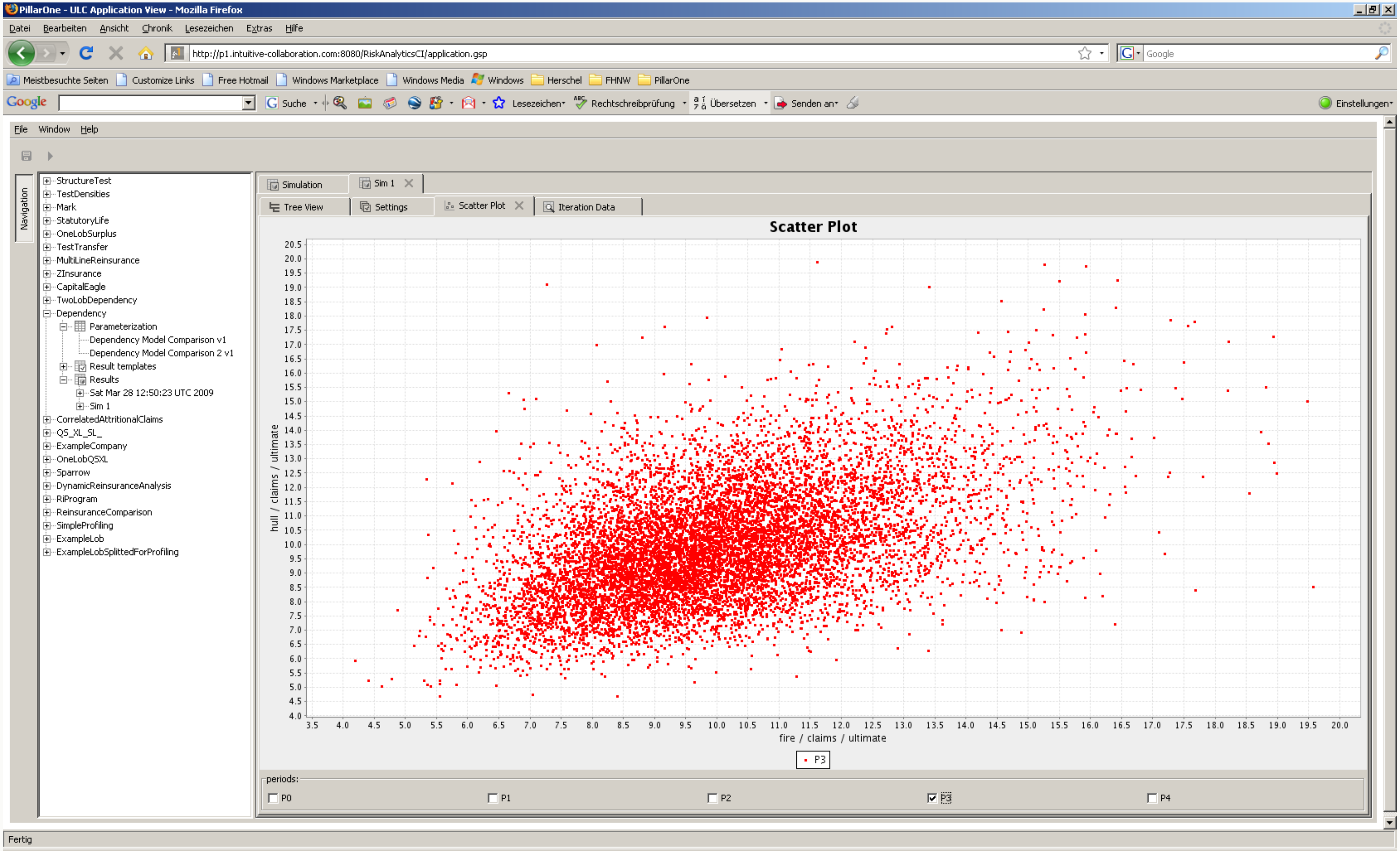
General Structure:

- Specified at model specification time.
- Utilities: Sensitivity functions to be defined and implemented.

Normal Copula



T-Copula (1)


PillarOne - ULC Application View - Mozilla Firefox
http://p1.intuitive-collaboration.com:8080/RiskAnalyticsCI/application.gsp
Simulation
Sim 1
Tree View
Settings
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Iteration Data
Scatter Plot
hull / claims / ultimate
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periods:
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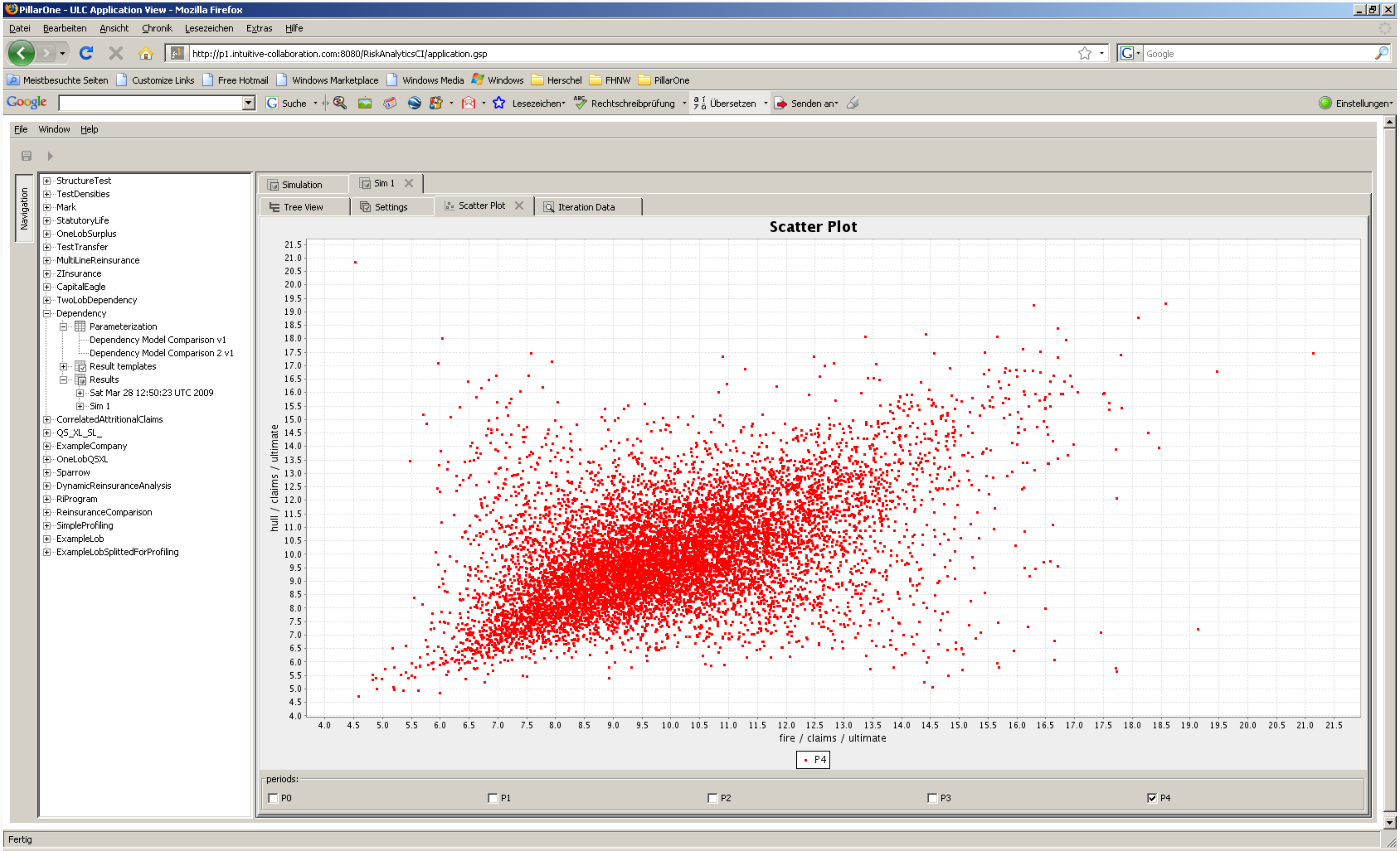
T-Copula (2)


PillarOne - ULC Application View - Mozilla Firefox
Scatter Plot
hull / claims / ultimate
fire / claims / ultimate
P4
periods:
P0
P1
P2
P3
P4

Example 2: Dependency of Aggregate Claims on Claims Origin